Boundedly-Rational Expectations and Macroeconomic (In-)Stability

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Introduction

- The recent financial crisis led to severe criticisms to the predominant DSGE approach (see e.g. Colander et al. 2009)
- One of the key criticism points concern the assumption of rational expectations, a main building block of mainstream macroeconomic models
- Alternative view: Keynes' (1936, pp. 161-162) "animal spirits":

> *Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits – a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.*

- As pointed out, e.g. by De Grauwe and Grimaldi (2006), Efficient Markets Rational Expectations models seem incompatible with important stylized facts of financial markets as well as the occurrence of speculative bubbles, herding behavior and currency runs.
- In contrast, behavioral "non-rational" models, i.e. models that feature economic agents with heterogenous beliefs, attitudes or trading schemes, seem much more successful in this task; see, for example, Frankel and Froot (1987), Allen and Taylor (1992), Cheung and Chinn (2001) and Manzan and Westerhoff (2007).

- Alternative approach in the finance literature: Heterogeneous agents with "boundedly rational" beliefs (Frankel and Froot (1990 AER), Allen and Taylor (1992 JIMF), De Grauwe and Grimaldi (2005 JEDC) and Manzan Westerhoff (2007 JEBO), among others)
- However: Most of these models focus solely on the financial markets!
- In recent times, heterogeneous expectations are being incorporated in a new generation of macroeconomic models, see e.g. Branch & McGough (2009, JEDC), De Grauwe (2011, ET), Proaño (2011, JEBO), Proaño (2013, MS), etc.
- In this models, the importance of rule-of-thumb behavior for macroeconomic stability is investigated

- In the undergraduate macro IS-LM model, for instance, private consumption is assumed to be a linear function of current income,

$$C_t = cY_t, \quad 0 < c < 1$$

- What if we knew that $Y_{t+1} > Y_t$? How would this affect private consumption today?
- Most decisions of the economic agents have an important forward looking element!
- Since the future is unknown, agents form *expectations* about future variables, which may, or may be correct.

The expectations formation process is intrinsically linked to

- the information set available to the economic agents at the time when expectations are formed, and
- the economic agents' degree of rationality and thus their capability to process that information.

- Consider for instance the Samuelson (1939) investment accelerator model:

$$\begin{aligned} C_t &= a + bY_{t-1} \\ I_t &= v(Y_{t-1} - Y_{t-2}) \\ E_t &= C_t + I_t \\ Y_t &= E_t \end{aligned}$$

- Straightforward substitution leads to a second-order difference equation in Y_t

$$Y_t = a + (b+v)Y_{t-1} - vY_{t-2}$$

$\Longrightarrow$ The exact v is crucial for the dynamic stability of the economy!

- What is the assumed information set in this model?

  $\Longrightarrow$ Apparently, firms do not observed contemporaneous (time t) variables, but just past ($t-1$, $t-2$) variables.

- What is the degree of rationality?

  $\Longrightarrow$ Simple rule of-thumb

Expectations in the Neoclassical Synthesis Framework

- During the 1950s and 1960s, the specification of expectations, and especially inflation expectations, in models of the Neoclassical Synthesis was undertaking through a static or an adaptive expectations formation scheme
- The dynamic version of the AS-AD model consists of the following equations

$$
\begin{aligned}
y_t &= b - b_r r_t && \text{Aggregate Demand} && (1)\\
r_t &= i_t - \pi^e_{t+1,t} && \text{Fisher equation} && (2)\\
\pi_t &= \pi^e_{t,t-1} + \phi(Y_t - \bar{Y}_t) + \nu_t && \text{Phillips Curve} && (3)\\
i_t &= k_y y_t - k_m(m_t - p_t) && \text{(Inverted) LM-equation} && (4)
\end{aligned}
$$

and

$$\pi^e_{t,t-1} = \pi^e_{t-1} + \lambda(\pi_{t-1} - \pi^e_{t-1}) \qquad (5)$$

i.e. adaptive inflation expectations.

Note on expectations

- π_t^e is the expected rate of inflation for period t
- The expectation was formed in period $t-1$.
- Expectations are backward-looking in this model.
- Adaptive expectations

$$\pi_t^e = \pi_{t-1}^e + \lambda(\pi_{t-1} - \pi_{t-1}^e)$$

- Correction of expectations error
- Special case $\lambda = 1$

$$\pi_t^e = \pi_{t-1}$$

$\Longrightarrow$ static expectations

- What is the assumed information set in this model?

  $\Longrightarrow$ Agents use only past values of the inflation rate to form their expectations

- What is the degree of rationality?

  $\Longrightarrow$ Simple rule of-thumb

Steady State

Endogenous variables: y, i , r, π, and π^e

- In steady state $\pi^e_t = \pi^e_{t-1} = \pi^{e*}$
- Hence $0 = \lambda(\pi_{t-1} - \pi^e_{t-1}) \Longrightarrow \pi^* = \pi^{e*}$

  $\Longrightarrow$ Expected inflation must be equal to actual inflation
- From Phillips curve: $0 = \theta(y_t - \bar{y}) \Longleftrightarrow y^* = y$
- The steady state real interest rate $r^* = \frac{b - \bar{y}}{b_r}$
- The steady state nominal interest rate $i^* = r^* + \pi^*$
- From $i_t - i_{t-1} = k_y(y_t - y_{t-1}) - k_m(m_t - m_{t-1} - (p_t - p_{t-1}))$, it holds

$$\pi^* = p_t - p_{t-1} = m_t - m_{t-1} = \hat{m}_t$$

$\Longrightarrow$ What about the dynamic behavior of the model?

Dynamic Analysis

We can reduce the model to the following single difference equation in deviations of the output level y_t from the steady state level $\bar{y}$, i.e. $\tilde{y}_t = y_t - \bar{y}$:

$$\tilde{y}_t + a\tilde{y}_{t-1} + b\tilde{y}_{t-2} \quad = \quad 0$$

with $a = [1 + \Phi(1 - \phi_1 \lambda\theta)]$, $b = \Phi$, $\Phi = \frac{1}{1+\theta(\phi_1 - \phi_2\lambda)}$, $\phi_0 = \frac{b}{1+b_r k_y}$, $\phi_1 = \frac{b_r k_m}{1+b_r k_y}$, and $\phi_2 = \frac{b_r}{1+b_r k_y}$, and

$$\bar{y} = [1 + \Phi(1 - \phi_1 \lambda\theta)]\bar{y} - \Phi\bar{y} + \Phi\phi_1\lambda\theta\bar{y}$$

Two questions:

1. When are there oscillations?
2. Are they damped or explosive (is the system stable)?

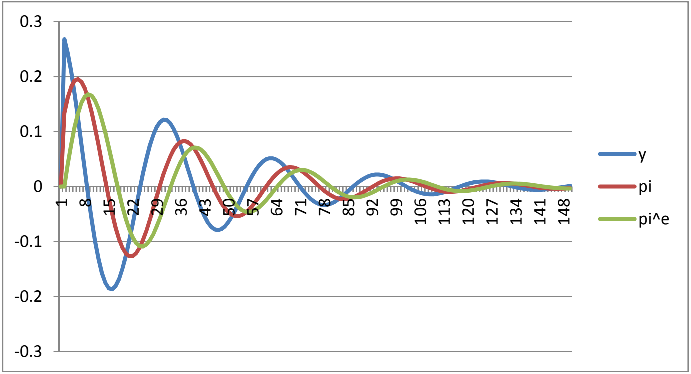


Figure: Dynamic adjustment after a one-time unit money supply shock ($b = 1$, $b_r = 0.6$, $k_y = 0.1$, $k_m = 0.5$, $\theta = 0.5$, $\lambda = 0.3$)

The “Rational Expectations” Revolution

- Due to the inflation shocks in the 1970s and the rapid increase in inflation, adaptive expectations specifications became increasingly unrealistic, as they implied that agents were computing “wrong” expectations *in a systematic manner*.
- Much earlier, John Muth (1961) proposed a particular expectations formation scheme which he labeled “rational expectations”
- Interestingly, he wrote in the same article the following:

  “At the risk of confusing this purely descriptive hypothesis with a pronouncement as to what firms ought to do, we can such expectations “rational”.”

Rational Expectations in a Nutshell

- Rational agents will use all the information available to them (the one contained in their information set) in the most efficient manner.
- Information set available at the beginning of period t:

$$\Omega_t \equiv \{\underbrace{y_{t-1}, y_{t-2}, \ldots, i_{t-1}, i_{t-2}, \ldots \pi_{t-1}, \pi_{t-2}, \ldots, \pi^e_{t-1}, \pi^e_{t-2}, \ldots}_{(a)}; \underbrace{a, b}_{(b)}; \underbrace{\nu_t \sim N(0, \sigma^2)}_{(c)}\}$$

(a) Agents do not forget (relevant) past information.
(b) Agents know the parameters of the model.
(c) Agents know the stochastic process of the shocks.

- REH in mathematical form: $x^e_t = E\left(x_t \mid \Omega_t\right) \equiv E_t x_t$, where we use the shorthand notation E_t to indicate that the expectation is conditional upon information set Ω_t.
- Then: the subjective expectation (x^e_t) coincides with the objective expectation conditional on the information set of the agent.

Implications (in the previous model)

Subjective expectations are model-consistent, i.e. $E_t(\pi_t) = \pi_t$.

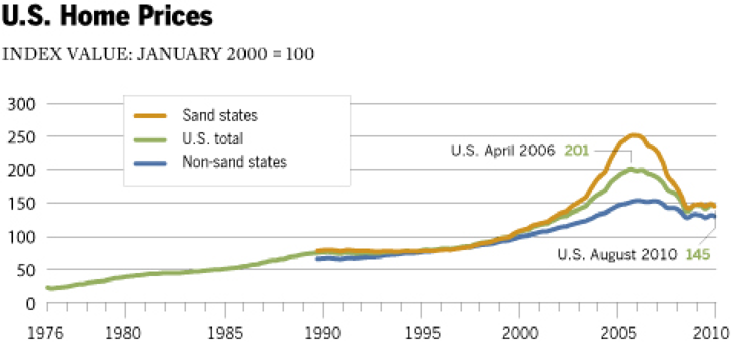
$\Longrightarrow$ Inflation is correctly anticipated on average!

$\Longrightarrow$ Deviation of y_t from $\bar{y}_t$ are only due to unforeseeable shocks

Implications (more broadly)

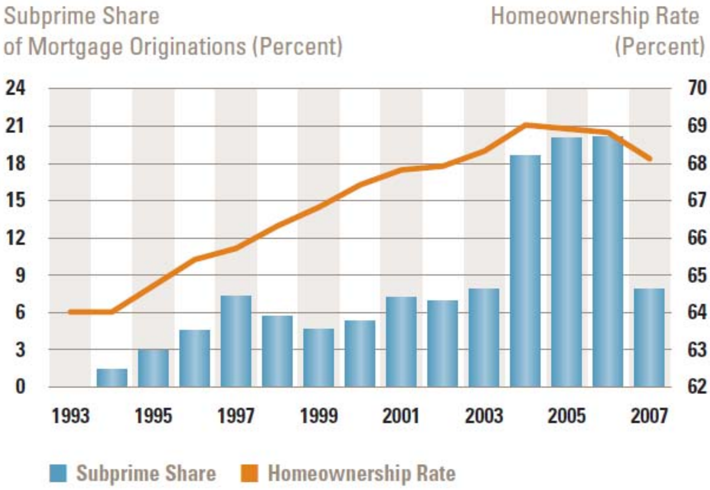
- If all agents know about the financial solvency of the other agents and understand the functioning of the economy, the allocation of resources in the economy should be efficient.
- In other words: If all agents maximize their utility/profits in a consistent manner with their intertemporal budget constraints and possess similar/same information sets, allocations should be optimal and Ponzi games ruled out.

The 2007 Subprime Crisis



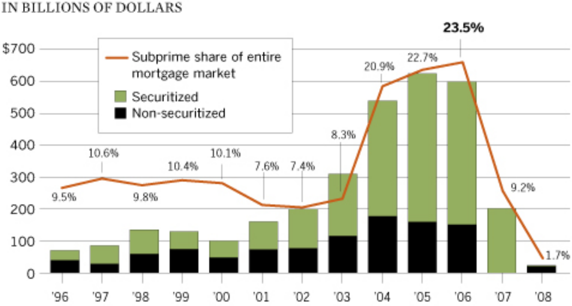
NOTE: Sand states are Arizona, California, Florida, and Nevada.

SOURCE: CoreLogic and U.S. Census Bureau: 2007 American Community Survey, FCIC calculations


Subprime Share
of Mortgage Originations (Percent)
Homeownership Rate
(Percent)
24
21
18
15
12
9
6
3
0
70
69
68
67
66
65
64
63
62
1993
1995
1997
1999
2001
2003
2005
2007
Subprime Share
Homeownership Rate

Subprime Mortgage Originations

In 2006, $600 billion of subprime loans were originated, most of which were securitized. That year, subprime lending accounted for 23.5% of all mortgage originations.



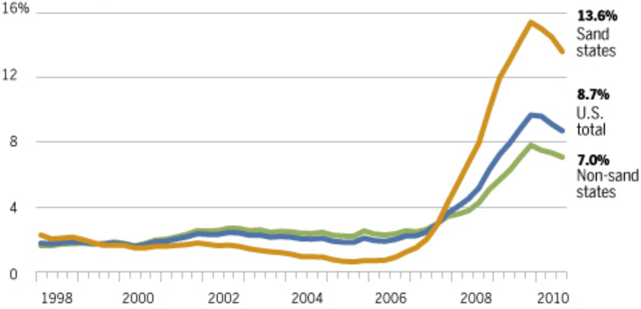
NOTE: Percent securitized is defined as subprime securities issued divided by originations in a given year. In 2007, securities issued exceeded originations.

SOURCE: Inside Mortgage Finance

Mortgage Delinquencies by Region

Arizona, California, Florida, and Nevada—the “sand states”—had the most problem loans.

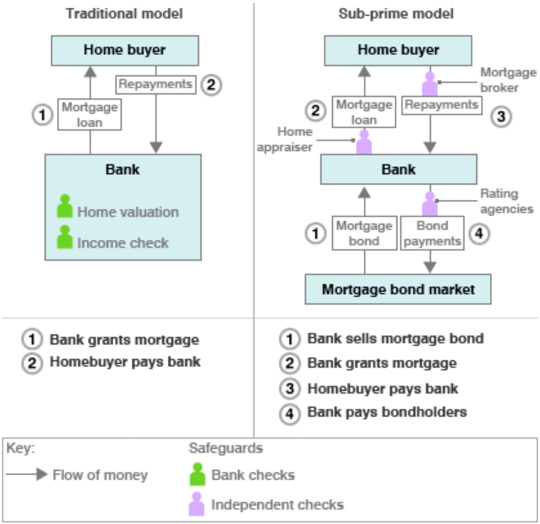
IN PERCENT, BY REGION



NOTE: Serious delinquencies include mortgages 90 days or more past due and those in foreclosure.

SOURCE: Mortgage Bankers Association National Delinquency Survey

- The recent financial crisis can be traced back to
  - Overly optimistic expectations concerning future house prices (and the feasibility of Ponzi games)
  - Wrong assessment of loan risk by banks
  - Lack of proper understanding of the limits of securitization concerning the diversification of risk


Traditional model
Home buyer
Repayments
2
1
Mortgage loan
Bank
Home valuation
Income check
Sub-prime model
Home buyer
Mortgage broker
2
Mortgage loan
Repayments
3
Home appraiser
Bank
Rating agencies
1
Mortgage bond
Bond payments
4
Mortgage bond market
1 Bank grants mortgage
2 Homebuyer pays bank
1 Bank sells mortgage bond
2 Bank grants mortgage
3 Homebuyer pays bank
4 Bank pays bondholders
Key:
Flow of money
Safeguards
Bank checks
Independent checks

The Proaño (2013, MS)Model

Sequence of Events

- At the beginning of a period t, the FX market participants form their forecasts of the nominal exchange rate at $t + 1$ on the basis of the information set containing macroeconomic data generated up to $t - 1$. Independently, the domestic monetary authorities set the nominal interest rate on the basis of the same information.
- Then, given the perfect capital mobility between the domestic economy and the rest of the world and the trading of the FX market participants on the basis of (possibly) different forecasts, the nominal exchange rate level of period t adjusts so that the Uncovered Interest Rate Parity (UIP) holds at the market level.
- Finally, the real variables output and inflation in the domestic economy are determined.

According to the "fundamentalist" forecasting rule, the expected log nominal exchange rate at $t+1$ is given by

$$E_t^f s_{t+1} = s_{t-1} + \beta_s^f (f_{t-1} - s_{t-1}), \tag{6}$$

where f_{t-1} represents the (log) fundamental nominal exchange rate at time $t-1$ and $\beta_s^f > 0$ a scaling factor linked with the speed of adjustment of the log nominal exchange rate towards its long-run equilibrium level f

$$f_t = p_t - p_t^* \tag{7}$$

Inserting this expression in eq.(6) delivers

$$\begin{aligned} E_t^f s_{t+1} &= s_{t-1} + \beta_s^f (p_{t-1} - p_{t-1}^* - s_{t-1}) \\ &= s_{t-1} - \beta_s^f (\eta_{t-1}) \end{aligned} \tag{8}$$

where η_t is the log of the real exchange rate $\mathcal{N} = SP^*/P$ at time t and $\eta_o = 0$ its PPP-consistent level.

By contrast, according to the “chartist” forecasting rule, it is assumed that the respecting expected log nominal exchange rate at $t+1$ is given by

$$E_t^c s_{t+1} = s_{t-1} + \beta_s^c \Delta s_{t-1}, \tag{9}$$

With (eventually) different expectations concerning the future development of the nominal exchange rate resulting from the two behavioral forecasting rules just described, the last-period earnings of investing one unit of domestic currency in the foreign currency depend of course on whether a nominal appreciation ($\Delta s_{t-1} < 0$) or a depreciation ($\Delta s_{t-1} > 0$) took actually place between the periods $t-2$ and $t-1$, that is

$$\psi^j_{t-1} = [S_{t-1}(1 + i^*_{t-1}) - (1 + i_{t-1})S_{t-2}] \operatorname{sgn}[E^j_{t-2}\Delta s_{t-1}] \quad j = c, f \tag{10}$$

with

$$\operatorname{sgn}[\, E^j_{t-2}\Delta s_{t-1} \,] = \begin{cases} 1 & \text{for } E^j_{t-2}\Delta s_{t-1} > 0 \\ 0 & \text{for } E^j_{t-2}\Delta s_{t-1} = 0 \\ -1 & \text{for } E^j_{t-2}\Delta s_{t-1} < 0 \end{cases}$$

At every t, the share of FX traders using the fundamentalist forecasting rule (the so-called "market mood") is determined by

$$\omega_t = \frac{\exp[\gamma(\psi^f_{t-1} - \sigma^2_{f,t-1})]}{\exp[\gamma(\psi^f_{t-1} - \sigma^2_{f,t-1})] + \exp[\gamma(\psi^c_{t-1} - \sigma^2_{c,t-1})]} \tag{11}$$

with

$$\lim_{\psi^f_{t-1}\to\infty} \omega_t = 1 \quad \text{and} \quad \lim_{\psi^f_{t-1}\to 0} \omega_t = 0,$$

and

$$\sigma^2_{j,t-1} = (E^j_{t-2}S_{t-1} - S_{t-1})^2 \quad j = c, f,$$

being the last period's squared forecast error of the behavioral forecasting rule j, and γ measuring the sensitivity with which traders revise their choice of the forecasting rules (a higher γ implying a stronger reaction to the profitabilities differentials between the two rules).

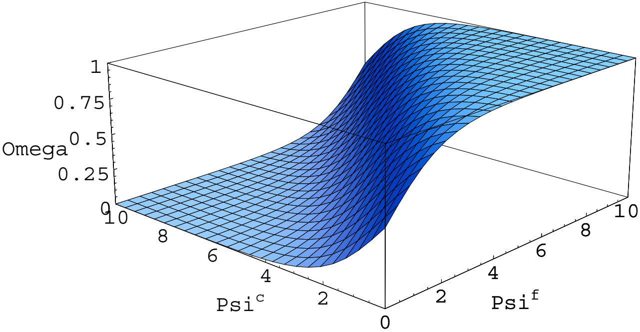


Figure: The ω_t function

On the basis of the expressions for $E_t^f s_{t+1}$ and $E_t^c s_{t+1}$ given by eqs. (6), (8), and (11), respectively, the market expectation of the log nominal exchange rate at $t+1$ is simply the weighted average of the two expected nominal exchange rates, that is

$$\begin{aligned} E_t^m s_{t+1} &= \omega_t E_t^f s_{t+1} + (1-\omega_t) E_t^c s_{t+1} \\ &= s_{t-1} - \omega_t \eta_{t-1} + (1-\omega_t)\Delta s_{t-1}. \end{aligned} \tag{12}$$

with ω_t given by eq.(11).
According to UIP:

$$s_t = i_t^* - i_t + E_t^m s_{t+1}. \tag{13}$$

or

$$s_t = i_t^* - i_t + s_{t-1} - \omega_t \eta_{t-1} + (1-\omega_t)\Delta s_{t-1}. \tag{14}$$

By subtracting s_{t-1} from both sides, we obtain the following behaviorally founded law of motion for the log nominal exchange rate

$$\begin{aligned} \Delta s_t &= i_t^* - i_t + E_t^m s_{t+1} \\ &= i_t^* - i_t - \omega_t \eta_{t-1} + (1-\omega_t)\Delta s_{t-1}. \end{aligned} \tag{15}$$

The Macroeconomy

- Standard open-economy AD-relationship

$$y_t = \alpha_y y_{t-1} - \alpha_{yr}(i_{t-1} - \pi_t - (i_o - \pi_o)) + \alpha_{y\eta}\eta_{t-1} \tag{16}$$

- Standard backward-looking Phillips Curve

$$\pi_t = \alpha_{\pi y} y_{t-1} + \alpha_\pi \pi_{t-1} \tag{17}$$

- The log real exchange rate:

$$\begin{aligned} \Delta\eta_t &= \Delta s_t + \bar{\pi}^* - \pi_t \\ &= i_t^* - i_t - \omega_t \beta_s^f \eta_{t-1} + (1-\omega_t)\beta_s^c(\Delta s_{t-1}) + \bar{\pi}^* - \pi_t. \end{aligned} \tag{18}$$

with

$$\omega_t = \frac{\exp[\gamma(\psi_{t-1}^f - \sigma_{f,t-1}^2)]}{\exp[\gamma(\psi_{t-1}^f - \sigma_{f,t-1}^2)] + \exp[\gamma(\psi_{t-1}^c - \sigma_{c,t-1}^2)]}$$

Monetary Policy

Concerning monetary policy, the following general specification for the domestic nominal interest rate

$$i_t = \phi_i i_{t-1} + (1-\phi_i)\left[i_o + \phi_\pi(\pi^c_{t-1} - \pi_o) + \phi_y \, y_{t-1} + \phi_s \Delta s_{t-1}\right] \tag{19}$$

is formulated, with

$$\pi^c_t = (1-\xi)\pi_t + \xi\pi^m_t = (1-\xi)\pi_t + \xi(\pi^*_t + \Delta s_t),$$

defining CPI inflation, $\pi^m_t = \pi^* + \Delta s_t$ being the domestic-currency inflation of foreign goods and ξ being the share of imported goods in the CPI basket.

A monetary policy rule with a PPI inflation target, an output gap target and an interest rate smoothing term can be obtained by setting $\xi = 0$ and $\phi_s = 0$, so that

$$i_t = \phi_i i_{t-1} + (1-\phi_i)\left[i_o + \phi_\pi(\pi_{t-1} - \pi_o) + \phi_y\, y_{t-1}\right].$$

In contrast, a flexible CPI inflation targeting without interest rate smoothing results when $\phi_i = 0$ and $\phi_s = 0$, namely

$$i_t = i_o + \phi_\pi(\pi^c_{t-1} - \pi_o) + \phi_y\, y_{t-1},$$

and a strict nominal exchange rate targeting without interest rate smoothing can be expressed by $\phi_\pi = 0$, $\phi_i = 0$ and $\phi_y = 0$ as

$$i_t = i_o + \phi_s \Delta s_{t-1}.$$

In the following we discuss the simulation results based on the following parameter values:

Table: Parameter Values

Output Gap	Phillips Curve	Monetary Policy	FX Markets
$\alpha_y = 0.9$	$\alpha_{\pi y} = 0.24$	$\phi_i = 0.7$	$\beta_s^f = 1/6$
$\alpha_{yr} = 0.1$	$\alpha_\pi = 0.8$	$\phi_\pi = 1.5$	$\beta_s^c = 1.25$
$\alpha_{y\eta} = 0.01$		$\phi_y = 0.5$	$\gamma = 10$

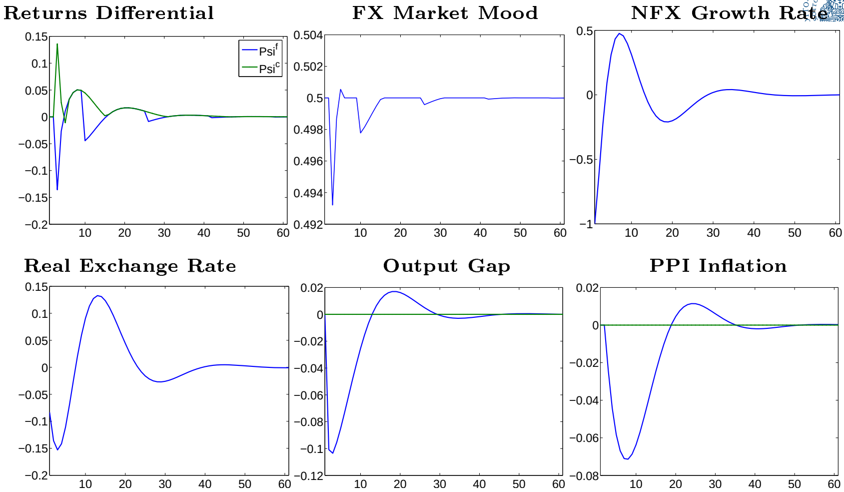


Figure: Dynamic responses of FX markets and real economy to a one-time domestic monetary policy shock

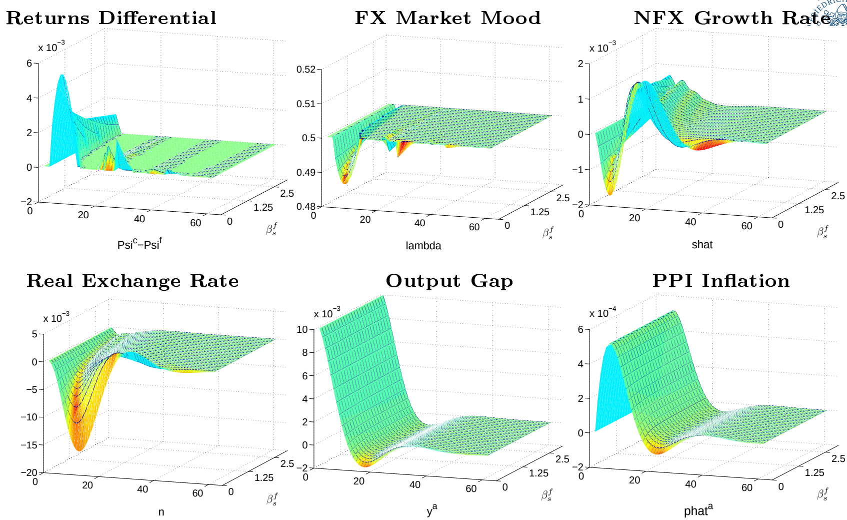


Figure: Dynamic responses of FX markets and real economy to a one-time aggregate demand shock for varying values of $\beta_s^f \in (0, 1.5)$ (with $\beta_s^c = 0.5$)

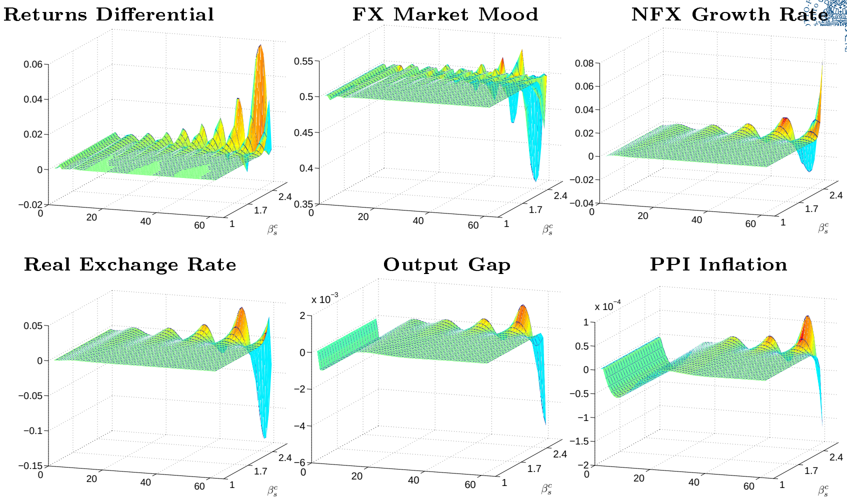


Figure: Dynamic responses of FX markets and real economy to a one-time domestic monetary policy shock for varying values of $\beta_s^c \in (0, 2.4)$ (with $\beta_s^f = 0.5$)

Key Insights

- Chartist expectations rules are destabilizing for the whole macroeconomy, and not only for the financial markets
- Fundamentalist expectations rules stabilizing
- The relative importance of both types of rules depend on their *profitability*

Concluding Remarks

- Agents have limited information sets and/or cognitive capabilities to process all the information available to them in reality
- They use rule-of-thumb rules instead, which can lead to market instability
- These issues need to be incorporated in macroeconomic models not as exceptions, but as the baseline scenario